

Managerial Strategic Investment with Agency and Competition in a Real Options Framework*

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Abstract

This paper examines the investment behavior of a managerial firm facing competition by developing an investment timing model within the real option exercise game framework under incomplete information. The particular research question of concern is whether the competition serves as an incentive mechanism for the agency problem. Product market competition is modeled in a full preemption fashion in the sense that the first mover captures the whole market and the second mover's option to invest becomes worthless. The delegation of investment decision to a manager creates an agency conflict since the true costs (or quality, equivalently) of the underlying project is observed privately by the manager which gives her the scope for diverting part of the cash flows for private benefits. Thus, an optimal contract has to be designed which induces the agent to truthfully reveal the project's costs and exercise the option at a strategically optimal trigger level. Our results indicate that while competition tends to induce (over-) early-investment for both types of the project, the agency problem calls for delaying the investment for the high cost (or low quality) project, with the overall effect being dependent on the relative importance of preemption threat to the agency conflict. Accordingly, the existence of preemption threat can mitigate the inefficiency stemming from agency conflict for the high cost project. Furthermore, competition provides additional incentives to the manager for truth-telling and as a result allows the owner to provide less (informational) rents to the manager. Finally, allowing for positive correlation between the competing firms' underlying project costs has two consequences: First, while the amount of investment timing adjustment required due to competition decreases for the high cost project, the same increases for the low cost (or high quality) project. Second, the presence of correlation provides (also) additional incentives to the manager for truth-telling and it suppresses the distortion in the high cost project's exercising trigger that originally stems from the agency problem.

Keywords: Investment timing, Agency, Contracting, Competition, Preemption, Incomplete information

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1 Introduction

The real options approach to investment has substantially altered the way how we think about irreversible investment opportunities. McDonald and Siegel (1986) were the first to formally show that optimal investment rules deviate from a simple NPV criterion and become “*an irreversible investment should only be undertaken if its expected payoff exceeds the sum of its cost and the value of waiting to invest*”. Moreover, due to its option-like characteristics and by a standard convexity argument, an increase in the uncertainty surrounding an investment’s payoff raises its value and also the value of waiting, thereby delays the investment. The standard real options approach is well summarized in Dixit and Pindyck (1994) and Trigeorgis (1996).

In the standard real options approach to investment under uncertainty, agents formulate optimal exercise strategies in isolation and ignore competitive interactions. However, in many real-world cases, exercise strategies cannot be determined separately, but must be formed as part of a strategic equilibrium. For instance, if firms fear preemption then the option to wait might become less valuable. Therefore, to understand investment in industries with competitive pressure, a game-theoretic analysis is called for¹. Some recent contributions in the literature are as follows: Grenadier (2002), Botteron, Chesney, and Gibson-Asner (2003), Lambrecht and Perraudin (2003), Paxson and Pinto (2003), Miltersen and Schwartz (2004), Murto (2004), Pawlina and Kort (2006), Smit and Trigeorgis (2006) Miltersen and Schwartz (2007), Anderson, Friedman, and Oprea (2010) and Graham (2011).

Moreover, both the standard real options approach to investment and the simple NPV rule do not account for information asymmetries and agency conflicts. It is often assumed that the option’s owner makes the exercise decision, i.e. no agency conflicts are allowed in the standard real options models. In most corporations, however, shareholders delegate the investment decision to managers, taking advantage of managers’ expertise and knowledge. In such decentralized settings, both hidden information (e.g., managers are better informed than owners about projected cash flows, the level of competition, the firm’s own and competitor firms’ investment opportunities) and hidden actions (e.g., unobserved managerial effort, empire building) are likely to exist. A number of papers, particularly in the corporate finance literature, examine how corporate investment is influenced by problems of

¹ See Boyer, Gravel, and Lasserre (2004) for a recent useful review on the main contributions to the joint analysis of real options and strategic competition.

asymmetric information and agency². Agency problems can be mitigated partially or fully by several practical mechanisms, which is the main theme of the contracting literature. The standard capital allocation literature provides predictions on whether firms over- or under-invest relative to the first-best no agency benchmark. In the real options literature, where investment timing is of interest, they translate into early- or delayed-investment³. The real option and traditional contracting (due to agency) literatures had remained separate (or combined limitedly) for a long time because of technical difficulties. For instance, real options are best understood in a continuous-time framework, while agency and contracting problems have traditionally been studied in discrete-time. Moreover, these models used very stylized setups, typically with two or three periods, that do not correspond to the standard models of investment used elsewhere in economics. Notice that, while one possibility is the agency issues arising between managers and shareholders⁴, similar issues could exist between stockholders and bondholders or outside investors⁵.

Do we observe both over- and under-investment in the (irreversible and flexible) investment decision of a managerial firm which is competing in the product market under preemption threat? Can competition serve as an incentive mechanism and allow the owner to decrease the (informational) rents extracted by the manager? Can such competition mitigate the inefficiency stemming from agency conflict? What consequences does allowing for positive correlation between the competing firms' underlying projects costs have, both on the competition and the agency problem? These are the main questions addressed in this study.

The core model in this paper can be described as follows: The firm owns a single project to invest and is flexible to choose its timing. Moreover, this investment decision is irreversible⁶. The investment decision is delegated to a manager, who privately observes the firm's own underlying project costs⁷. An agency conflict then emerges since, for instance,

² See [Stein \(2001\)](#) for a useful summary on the impact of information and agency problems on investment behavior.

³ Early-investment can be interpreted as over-investment, in a way the overall project life is longer eventually.

⁴ [Grenadier and Wang \(2005\)](#), [Philippon and Sannikov \(2007\)](#), [Lambrecht and Myers \(2008\)](#), [Shibata and Nishihara \(2010\)](#) and [Gryglewicz and Hartman-Glaser \(2014\)](#) are recent studies that analyze the impact of such agency conflicts within a real option framework.

⁵ For instance [Mauer and Ott \(2000\)](#), [Morellec \(2004\)](#), [Mauer and Sarkar \(2005\)](#), [Morellec and Schürhoff \(2011\)](#), [Grenadier and Malenko \(2011\)](#) and [Bouvard \(2012\)](#) examine the impact of agency conflicts arising between stockholders and bondholders or outside investors on firm value using the real options approach.

⁶ It requires an initial expenditure (sunk cost) that should be paid by the time of investment.

⁷Note that, we could consider instead a manager privately observing the firm's own underlying project

the manager of a low cost (or high quality) project could claim to have a high cost (or low quality) project and divert part of the cash flows for her private benefits. Meanwhile, the firm is competing in the product market in a full preemption fashion⁸. Due to the incomplete information setting, the firm has only a belief on the exercise trigger of its competitor. Further, we allow for positive correlation between both firms' underlying project costs. If there exists correlation, by observing privately the realization of the own underlying project characteristics the manager receives additional (private) information about its rival's behavior and updates her beliefs accordingly. To induce truth-telling the owner designs and offers a contract to the manager specifying a set of recommended exercise triggers and associated wages, one for each possible realization of project cost. Finally, upon exercise, the owner receives the value of the underlying project, pays the exercise price and the manager's exercise wage. On the other hand, upon preemption, the owner does not receive any value but may pay the manager a *preemption wage*.

As a first step, we derive the optimal investment behavior of an *entrepreneurial* firm under fear of preemption. We refer to these results as the first-best benchmark model results. Second, we derive the optimal investment behavior of a *managerial* firm under fear of preemption; in order to gain a first insight we assume *no correlation* and analyze the optimal wage scheme, i.e. derive the expressions for the exercise trigger levels and corresponding wages for each type of project. Comparing the exercise trigger levels with the first-best benchmark results, we find that, as usual in adverse selection models, there is no efficiency distortion at the top (low cost project)⁹, whereas the high cost project is exercised later in order to satisfy the incentive constraint for the manager of a low cost project. Thus, the relative position of high cost project's trigger level compared to the standard real options case¹⁰ very much depends on model parameters; in particular on the importance of preemption threat relative to the agency conflict¹¹. Consequently, both over- and under-investment can occur within the investment decision of a managerial firm under preemption threat. Notice that, competition can mitigate the inefficiency stemming

quality (high or low). During modeling we will show that this approach, while providing alternative insights, is mathematically equivalent. Throughout the analysis we use the cost and quality interchangeably; high quality project maps to low cost project, and low quality project maps to high cost project.

⁸ The first mover captures the whole market and the second mover's option to invest becomes worthless.

⁹ in the sense that the low cost project is exercised at the same level as it would be in a model without agency

¹⁰ with no agency and no competition

¹¹ Notice that in the limiting cases our results collapses into the ones of Grenadier and Wang (2005) when only agency conflicts exist and into the ones of Lambrecht and Perraudin (2003) when only competition exists.

from agency by bringing the high cost project’s exercising trigger back to the efficient trigger level¹². Furthermore, competition lets the gap between the triggers serve to provide additional incentives to the manager for truth-telling and as a result allows the owner to supply less (informational) rents to the manager. This is our first contribution to the real options literature.

Our other contribution to the literature lies in studying the effect of positive *correlation* between the competing firms’ underlying project values on competition and agency conflict. Introducing correlation has two interesting consequences within our extended real investment framework. First, regarding an *entrepreneurial* firm: In case of a low cost project, the early exercise of the competitor becomes more likely and the preemption threat grows. Hence, while the amount of investment timing adjustment required due to competition increases for the low cost project, the same decreases for the high cost project. Second, regarding a *managerial firm*: Remember that, competition itself lets the gap between the triggers serve to provide additional incentives to the manager for truth-telling. The correlation, in addition to competition, emphasizes this role of the gap between the triggers for providing additional incentives to the manager¹³. Moreover, the presence of correlation suppresses the distortion in the high cost project’s exercising trigger that originally stems from the agency problem.

The remainder of the paper is organized as follows. The next section presents the literature the paper is related to. Section 3 contains the model descriptions and analyzes. Particularly, Section 3.1 presents a *basic* model “a single *entrepreneurial* firm’s investment behavior” which serves as a building block for the following Section 3.2, where the *first-best benchmark* model “the *entrepreneurial* firm’s investment behavior affected by preemption fear” is introduced. Section 3.3 describes the setup of our *core principal-agent* model “optimal investment of a *managerial* firm threatened by *preemption*”, and allows as well for the existence of positive correlation between both firms’ costs, and analyzes it thoroughly. In Section 3.4 we discuss two major extensions; allowing for a continuous distribution of own project costs and establishing (Bayesian Nash) equilibrium. Section 3.5 discusses limitations of our *core principal-agent* model. Section 4 concludes. The Appendix contains the solution details of the optimal contracts and proofs of relevant propositions.

¹² We define the inefficiency as any deviation from the basic model optimal exercise triggers.

¹³and as a result, for allowing the owner to provide less (informational) rents to the manager

2 Related Literature

There are four major strands of literature that our paper relates to. The first one extends the real options framework to account for the agency conflicts between shareholders and managers, allowing the existence of moral hazard and/or adverse selection. The setting of our paper is most similar to that of [Grenadier and Wang \(2005\)](#) in this area. There are two main differences between their model and ours. They analyze a single firm without the fear of preemption. But, they allow in addition that the manager can influence probabilistically the quality (or cost, equivalently) of the project by initially exerting a costly and unobservable effort. They design an optimal contract that induces the manager first to exert high effort and second to reveal her private information on the actual project quality by choosing the appropriate investment timing. Similar to our work, they show that managers display greater inertia in their investment behavior, in that they invest later than implied by the no agency case. They find that the nature of the optimal contract depends explicitly on the relative severity of these two forces; in case one of them is highly severe, it dominates solely the contract since the other effect is automatically mitigated. The hidden action alone does not result in inefficiencies, while hidden information alone does. Eventually, in their setting the interaction between hidden information and hidden action could reduce the inefficiency in investment timing, compared with the setting in which hidden information is the only friction. In our model, the competition interacts with the hidden information and that has similar consequences in reducing the inefficiencies. Last, in their hidden information setting only under-investment is achieved, while in our model both under- and over-investment are possible.

The second strand of related literature analyzes how strategic behavior could be integrated with the contingent claims techniques employed in the real options literature. In particular, this area of research investigates the effect of (product market) competition on investment timing decisions. In some cases, firms do better by delaying until their competitors act first (attrition). In some situations, like in our model, a firm fears that a competitor may seize an advantage by acting first (preemption). The setting of our paper is most similar in this field to that of [Lambrecht and Perraudin \(2003\)](#). There exist mainly two differences between our model and theirs. First, in their model they do not allow for agency conflicts. Second, they are not accounting for possible correlations within the competitive environment which might alter, as in our model, the impact of competition¹⁴.

¹⁴ Within their simpler setting they construct and analyze a (Bayesian Nash) equilibrium under incom-

There is an interesting line of research on strategic behavior in the context of patent (or R&D) races, for which using a full preemption modeling approach is well justified. Along the literature of patent races, technological competition has been investigated first through stationary games under uncertainty by [Loury \(1979\)](#), [Dasgupta and Stiglitz \(1980\)](#), and [Lee and Wilde \(1980\)](#), and then through dynamic games under uncertainty but without explicit strategic interactions pioneered by [Reinganum \(1981\)](#) and [Reinganum \(1982\)](#) or through dynamic games with strategic interactions but without uncertainty pioneered by [Fudenberg, Gilbert, Stiglitz, and Tirole \(1983\)](#) and [Harris and Vickers \(1985\)](#). Finally, strategic interactions and technological uncertainties are combined within a dynamic structure, as in [Judd \(2003\)](#), [Grossman and Shapiro \(1987\)](#), and [Harris and Vickers \(1987\)](#). In [Loury \(1979\)](#), which is one of the pioneers in the patent race literature, two firms choose actively their own investment rates at which they develop the new technology, and the winner of the race is awarded a patent. First of all, even though increased investment might lead to quicker innovation, eventually there is wasteful duplication of effort. Next, competition makes the firms more eager to invest and as a result they may over-invest in the patent race relative to the scenario of joint monopoly. Moreover, the over-investment induced by competition erodes the aggregate value of the firms. The models discussed above lack the cash flow uncertainty component since the patent value is set as a constant, and hence, a real-options methodology has not been applied. Though, a recent work by [Weeds \(2002\)](#) studies a patent race model in the context of real options, where, in contrast to our model, the first mover firm's investment decision does not result in an immediate full preemption, but allows this firm to enter a probabilistic innovation phase for winning the patent race. Meanwhile, the second mover can still invest and enter the same innovation phase. Eventually, the firm who wins within this probabilistic structure gets the patent, preempts the other and receives the stochastic future cash flows. [Weeds \(2002\)](#) shows that strategic interactions between the firms can induce an additional investment delay under uncertainty. A useful analogy is the behavior of contestants in a long-distance race, who typically remain in a pack proceeding at a moderate pace for most of the distance, until near the end when one attempts to break away and the sprint for the finish begins. Notice that, the main difference between the patent races literature and our analysis is that, in this former literature there exists a phase where the firms are active to influence their successes in the technology development. Eventually, as [Loury \(1979\)](#) argues, patent races might induce over-investment, or for the case of [Weeds \(2002\)](#) they can induce delayed-

plete information in which two or more firms invest subject to threats of preemption from competitors.

investment. Our modeling approach, as well as [Lambrecht and Perraudin \(2003\)](#)'s, would fit better for firms which are competing on an existing patent according to their cost structures. And, we find that competition induces early-investment, which can translate into the over-investment outcome of the standard capital allocation literature.

Next, a rather classical strand of literature from which our paper adopts, links the severity of the principal-agent problem to the degree of competition in product markets. [Hart \(1983\)](#), [Scharfstein \(1988\)](#), [Hermalin \(1992\)](#), [Schmidt \(1997\)](#) and [Aggarwal and Samwick \(1999\)](#) consider whether product market competition induces managers to improve efficiency by increasing their supply of effort. In his influential work, [Hart \(1983\)](#) finds that competition can serve as an incentive mechanism in that it reduces managerial slack if there exists correlation between firms' costs. In our model, by allowing correlation we analyze whether a similar impact of competition on agency exists or not, but within a real investment decision framework. While [Hart \(1983\)](#) is one of the first to derive this result in a formal model, his analysis is restricted to perfect competition and hence avoids strategic interactions among competitors. Last, while [Hart \(1983\)](#) studies a hidden action problem, our model focuses on a hidden information problem. [Aggarwal and Samwick \(1999\)](#) separates from the other mentioned studies in the sense that [Aggarwal and Samwick \(1999\)](#) allow the compensation contracts in turn to influence (imperfect) competition in the product market. Their finding is that in order to soften the effects of product market competition it can be optimal, as considered in our model, to include rival firms' performance in the incentive scheme of the own manager. As a limitation, their results are derived under the assumption of linear incentive schemes.

Last, there exists a recently developing literature that studies interactions among private information, unobservable effort and competition for the investment timing problem of firms. One of them is [Maeland \(2010\)](#). In this model, an owner of some project needs an expert (an agent) to manage the investment of the project. There are two or more agents with private information about their respective cost of investing in the project. The project owner organizes an auction, in which the agents participate. The investment strategy, formulated as an optimal stopping problem, is delegated to the auction winner. An optimal compensation function is derived, which induces the winner to follow the investment strategy preferred by the project owner. It is shown in this study that private information increases the project owner's cost of exercising the option, which may lead to under-investment. Our model differentiates itself from this study in the sense that this

model rather studies a labor market (including outsourcing and suppliers) competition than product market competition. Consequently, the formulation of competition and some other driving forces of effects as well as implications in our model are substantially different from those of this study.

3 Model and Analysis

3.1 Basic Model (Optimal investment of a single *entrepreneurial* firm)

Model

In this subsection, we develop a basic model of a single *entrepreneurial* firm's investment behavior which serves as a building block in our subsequent analysis of an *entrepreneurial* firm's behavior affected by preemption fear.

We assume throughout the analysis that investors are risk neutral and can borrow and lend freely at a constant risk-free rate of interest, $r > 0$ ¹⁵.

The firm, acting over an infinite time horizon, owns a single project to invest and is flexible to choose its timing. Moreover, this investment decision is irreversible and results in future stochastic cash flows.

For the time being, we assume that the firm is *entrepreneurial*, i.e. the owner manages the firm, therefore at this stage there are no agency problems as there is no separation of ownership and control. Similarly, we assume that the firm is all-equity financed to rule out conflicts between the bondholders and the shareholders regarding the firm's investment decision.

Similar to Grenadier and Wang (2005), once investment takes place, the project generates

¹⁵ Note that, introducing risk aversion hardly alters the valuation analysis of the *entrepreneurial* firm under preemption threat, if one assumes sufficient completeness of markets and follows the risk-neutral asset valuation technique. In the agency context, with the limited-liability condition we achieve our investment inefficiency results, even under risk neutrality. Assuming managerial risk aversion itself would generate an investment inefficiency in this context. In order not to let this inefficiency interfere with our results and to keep our model parsimonious, when we model the *managerial* firm later in the analysis, we model both the owner and the manager to be risk neutral. An alternative way to allow the owner and the manager value payoffs differently, especially in the context of real options, is to relax the assumption that both the principal and the agent are equally patient and assign them different discount factors.

two sources of value. One portion is $P(t)$,¹⁶ while the other portion is θ' realizing at time zero¹⁷. The investment requires an initial expenditure (sunk cost), expressed as K , that should be paid by the time of investment. Therefore the investment payoff is $(P(t) + \theta' - K)$. This way of modeling is well documented within the existing contracting literature, including Grenadier and Wang (2005). From the mathematical point of view, without loss of generality, the same problem could be equivalently formulated as $P(t)$ to be the whole project value, and $\theta (= K - \theta')$ to be initial expenditure which has a negative component $(-\theta')$ that realizes at time zero. Then, the investment payoff is $(P(t) - \theta)$ ¹⁸.

Let the value $P(t)$ evolve as a geometric Brownian motion,

$$dP(t) = \alpha P(t)dt + \sigma P(t)dZ(t) \quad (1)$$

where α is the conditional expected percentage change in $P(t)$ per unit time, σ is the conditional standard deviation per unit time, and $dZ(t)$ is the increment of a standard Wiener process. This implies that the current value of the project is known, but future values are log-normally distributed with a variance that grows linearly with the time horizon. Let P_0 equal the value of the project at time zero. For convergence, we assume that $r > \alpha$. In this way, we also allow for an optimum investment decision timing to exist.

θ is random, however already realizes θ at time zero. θ could take on two possible values: θ_1 or θ_2 , with $\theta_2 > \theta_1$. We denote $\Delta\theta = \theta_2 - \theta_1$. One could interpret a draw of θ_1 as a low cost (or high quality) project and a draw of θ_2 as a high cost (or low quality) project. The probability of drawing a low cost project θ_1 equals q . For the time being, the owner himself observes the project cost at time zero .

Due to its nature and the setting described above, the firm's investment opportunity is

¹⁶For ease of presentation, we model the process $P(t)$ directly as the present value of cash flows. We could back up a step and begin with an underlying process for cash flows. However, if cash flows, $X(t)$, follow a geometric Brownian motion with a percentage drift of α , then the present value of expected future cash flows will also follow a geometric Brownian motion with the percentage drift of α , given that the technical condition $r > \alpha$ is satisfied so that during the present value calculation, the integration of the discounted cash flows over time does not explode. Particularly, $P(t) = \frac{X(t)}{r-\alpha}$.

¹⁷Note that, at this point the firm is *entrepreneurial* and might not be clear why there are separate portions. Later, for the case of a *managerial* firm, we will allow $P(t)$ to be observable and contractible to both the owner and the manager, while the other portion, θ' , will be privately observed only by the manager.

¹⁸Grenadier and Wang (2005) follows also the same approach, however for the rest of their analysis they keep using the θ' notation, and referring to it as an indication of the project quality. We will, instead, use θ throughout our analysis. But, while interpreting our results, we will benefit from both of the notations; high quality project maps to low cost project, and low quality project maps to high cost project.

equivalent to an American perpetual call option on a stock¹⁹. In a standard call option setting, exercise yields the difference between the value $P(t)$ of the underlying asset and the exercise price θ .

Analysis

The derivation of the firm's value and optimal investment policy is standard. To save space, we provide the solution and refer the interested reader to Dixit and Pindyck (1994) for further details.

Proposition 1 *Under the above assumptions, a realized θ and a predetermined arbitrary exercise trigger $P > \theta$, the value of the firm at time zero is^{20, 21}*

$$V(P_0, \theta; P) = \left(\frac{P_0}{P}\right)^\beta (P - \theta) \quad (2)$$

where $\beta = \frac{1}{\sigma^2} \left[-(\alpha - \frac{\sigma^2}{2}) + \sqrt{(\alpha - \frac{\sigma^2}{2})^2 + 2r\sigma^2} \right] > 1$. After observing θ , in order to maximize the firm value, the owner chooses the optimal exercise trigger level as,

$$P^* = \frac{\beta}{\beta - 1} \theta \quad (3)$$

Proofs of this and subsequent results appear in the Appendix.

The θ is the NPV trigger level for $P(t)$ and investing in the project at this level generates a *positive* net present value. $P^* > \theta$ is the trigger level obtained by using the real options approach and therefore investing at this trigger level generates the *maximized* expected net present value.

3.2 First-Best Benchmark Model (Optimal investment of an *entrepreneurial* firm threatened by *preemption*)

Model

In this subsection, we extend the basic model from the previous subsection and analyze the *entrepreneurial* firm's investment behavior under preemption fear. This analysis will

¹⁹ For such an option, optimal exercise time is when the underlying project value first reaches a constant (over time) trigger level.

²⁰ We assume that $P \geq P_0$.

²¹ Note that throughout the whole analysis we denote the trigger levels with P , without any subscript. It is different than the process itself, $P(t)$.

be the first-best benchmark when we later analyze the *managerial* firm's behavior under same conditions.

To model a threat of preemption, let us suppose that a firm i seeks an optimal investment policy (as already described in the previous section); however, another firm, labeled j , may invest first, in which case firm i loses any further opportunity to invest.²² In order to avoid that the option value is destroyed completely due to this fierce form of competition, we assume that the competitor's characteristics are not fully known to the firm.

Thus, to introduce incomplete information, we assume firm i conjectures that firm j invests when $P(t)$ first crosses some level P_j , and that P_j is an independent draw from a distribution $F(P_j)$ with continuously differentiable density $F'(P_j)$ on the support $(\underline{P}_j, \overline{P}_j)$. Note that, for simplicity we do not start with modeling a cost distribution for firm j , but we assume an exogenously given $F(P_j)$ ²³. We can think of $F(P_j)$ as the belief about the competitor's trigger level absent any further information. However, for instance, due to the existence of positive correlation between both firms' costs²⁴ it seems reasonable to assume that the realization of firm i 's cost θ carries some information about the distribution of firm j 's exercise trigger²⁵. Thus, by observing the realization of θ the owner²⁶ receives additional information about its rival's behavior and updates his beliefs accordingly as specified in the *conditional* distribution $F(P_j|\theta)$ which is also assumed to have a continuously differentiable density $F'(P_j|\theta)$ on the interval, $(\underline{P}_j, \overline{P}_j)$. Under positive correlation between the costs of the competing firms, it is expected there exists positive correlation also between the firm i 's own cost and firm j 's trigger level. We capture this by assuming that the distribution of P_j conditional on θ_2 first order stochastically dominates the one conditional on θ_1 , i.e. $F(P_j|\theta_1) \geq F(P_j) \geq F(P_j|\theta_2)$.²⁷ In order to rule out the case

²² Similar to [Lambrecht and Perraudin \(2003\)](#), we model competition within a *full* preemption setting, where there is no benefit of investing for the second mover. Milder outcomes would be obtained if *partial* preemption was allowed for. However, since our main results are more emphasized in the full preemption setting, and for simplicity, we restrict our analysis to this case. For partial preemption argument within real options exercise games cf. [Botteron, Chesney, and Gibson-Asner \(2003\)](#) and [Pawlina and Kort \(2006\)](#).

²³ One could step back and start with the competitor's cost distribution, instead of optimal trigger level distribution, and determine the (Bayesian Nash) equilibrium distribution endogenously. In Section 3.4 we will discuss this.

²⁴We assume that, correlation stems from common components in the project characteristics.

²⁵ For further justifications cf. [Hart \(1983\)](#).

²⁶ Note that, we are still in a setting where the firm is entrepreneurial, i.e. the owner manages the firm, therefore at this stage there are no agency problems as there is no separation of ownership and managerial control.

²⁷ Further in the analysis, when interpreting the results, we will assume hazard rate dominance, cf. section 3.3. This implies first order stochastic dominance.

where firm j 's trigger level is below or equal to the initial project value P_0 , i.e. to prevent that firm i might be preempted already at time zero, we assume $\underline{P}_j \geq P_0$. This implies $F(P_0|\theta) = 0$.

Analysis

Proposition 2 *Under the above assumptions, a realized θ and a predetermined arbitrary exercise trigger $P > \theta$, the value of the firm at time zero is²⁸,*

$$V(P_0, \hat{P}_0, \theta; P) = \left(\frac{P_0}{P}\right)^\beta (P - \theta) \left(\frac{1 - F(P|\theta)}{1 - F(\hat{P}_0|\theta)}\right) \quad (4)$$

where $\hat{P}_t := \sup\{P_\tau : -\infty \leq \tau \leq t\}$. After observing θ , in order to maximize the firm value, the owner chooses the optimal exercise trigger level given by,

$$P^{**} = \frac{\beta + \hat{h}(P^{**}|\theta)}{\beta + \hat{h}(P^{**}|\theta) - 1} \theta \quad (5)$$

where the adjusted hazard rate, $\hat{h}(P|\theta)$, is defined as P times the standard hazard rate, $h(P|\theta) := \frac{F'(P|\theta)}{1 - F(P|\theta)}$.

Note that, compared to Eq. (2), Eq. (4) has an additional term in the end which corresponds to the conditional probability that firm i will not be preempted. The case that firm i is preempted, i.e. loses any further opportunity to invest, is not explicitly reflected in Eq. (4) since firm i gets zero net present value in that situation. We can conclude that, the *entrepreneurial* firm value under *preemption* threat is lower than the single *entrepreneurial* firm value. Note also, $P^{**} < P^*$ indicating that the fear of preemption lowers the optimal exercise trigger level, which is in line with our intuition.

It will prove useful in future calculations to determine at this point the present value of one unit of currency received at the first moment that a predetermined arbitrary exercise trigger level P is reached. Denote this present value operator by the discount function $D(P_0; P)$. This is simply the case where $(P - \theta)$ is replaced by 1 in Eq. (2) and can be stated as,

$$D(P_0; P) = \left(\frac{P_0}{P}\right)^\beta \quad (6)$$

²⁸ We assume that $P \geq P_0$.

On the other hand the present value of one unit of currency received at the first moment that a predetermined arbitrary exercise trigger level P is reached and preemption has not occurred before, $D(P_0, \hat{P}_0, \theta; P)$ is simply the case where $(P - \theta)$ is replaced by 1 in Eq. (4) and can be expressed as,

$$D(P_0, \hat{P}_0, \theta; P) = \left(\frac{P_0}{P}\right)^\beta \left(\frac{1 - F(P|\theta)}{1 - F(\hat{P}_0|\theta)}\right) \quad (7)$$

Eq. (4) expresses the firm value at time zero given a realized θ . Thus, the expected value of the firm, before θ is realized and following ex post the optimal trigger policy is,

$$\begin{aligned} W^{**}(P_0, \hat{P}_0) &= qV^{**}(P_0, \hat{P}_0, \theta_1; P_1^{**}) + (1 - q)V^{**}(P_0, \hat{P}_0, \theta_2; P_2^{**}) \quad (8) \\ &= q \left(\frac{P_0}{P_1^{**}}\right)^\beta (P_1^{**} - \theta_1) \left(\frac{1 - F(P_1^{**}|\theta_1)}{1 - F(\hat{P}_0|\theta_1)}\right) \end{aligned}$$

$$+ (1 - q) \left(\frac{P_0}{P_2^{**}}\right)^\beta (P_2^{**} - \theta_2) \left(\frac{1 - F(P_2^{**}|\theta_2)}{1 - F(\hat{P}_0|\theta_2)}\right) \quad (9)$$

where $P_1^{**} := P^{**}(\theta_1)$ and $P_2^{**} := P^{**}(\theta_2)$.

3.3 Principal-Agent Model (Optimal investment of a *managerial* firm threatened by *preemption*)

Model

Suppose now that initially the principal (owner) delegates the investment decision to an agent (manager). We assume that, $P(t)$, the project value, is observable and contractible to both the owner and the manager, while θ is the initial expenditure which has a component privately observed only by the manager when it realizes at time zero²⁹. An agency conflict then emerges as the manager of a low cost project could claim to have high costs and divert $\Delta\theta$ for private benefits. We assume that both the agent as well as the principal are risk neutral (with common discount rate r) and further the manager is protected by limited liability. To induce truth-telling the owner offers the manager a contract specifying a set of recommended exercise triggers P and associated wages w , one for each possible

²⁹ Remember that, the hidden information problem in the model setting of Grenadier and Wang (2005) is that the underlying project's value contains a component that depends on the *project quality* that is only privately observed by the manager. Initially, we defined the model this way, and later without loss of generality we switched to an equivalent formulation where it is then the *project costs* having a component that is observed privately.

realization of costs. Notice, that we can focus on truth-telling contracts as the revelation principle applies to our standard hidden information problem. Hence, we assume that the manager truthfully reveals her type at the beginning³⁰. The contract therefore specifies wage payments contingent on the observable (and verifiable) project value P at the time of exercise. Moreover, it can depend on whether or not, and at which level of project value (denoted by P_j) firm i is preempted by firm j . Theoretically, for any possible exercise value P a wage $w(P)$, and for any possible preemption value P_j a wage $w(P_j)$ can be specified, provided that both $w(P) \geq 0$ and $w(P_j) \geq 0$.

Upon *exercise*, the owner receives the value of the underlying project, pays the exercise price θ and the manager's *exercise* wage $w(P)$. Sum of the manager's and owner's payoffs equals the payoff of the underlying option to invest. On the other hand, upon *preemption*, the owner does not receive any value but may pay the manager a *preemption* wage $w(P_j)$. The manager's payoff is simply determined by the contingent wage scheme $\{w(P), w(P_j)\}$. Given that θ has only two possible values, we only need to specify two exercise trigger/wage/preemption wage triples from which the manager can choose. In our analysis, we allow for the possibility of a pooling equilibrium in which these two exercise trigger/wage/preemption wage triples are equal to each other. However, as we will see during the analysis, this pooling equilibrium is always dominated by a separating equilibrium. Therefore, the owner offers a contract that promises a wage of w_1 if the option is exercised at P_1 and a wage of w_2 if the option is exercised at P_2 . In addition, if a manager initially indicates that he has privately observed θ_1 , he is offered a preemption wage of $w_1(P_j)$, otherwise a preemption wage of $w_2(P_j)$. The revelation principle ensures that a manager who privately observes θ_1 exercises at the P_1 trigger, and a manager who privately observes θ_2 exercises at the P_2 trigger, in case he is not preempted.

The owner's objective is to maximize his expected net payoff via its choice of the contract terms $w_1, w_2, w_1(P_j), w_2(P_j), P_1, P_2$ ³¹. Thus, the owner solves the optimization problem,

$$\max_{w_1, w_2, w_1(P_j), w_2(P_j), P_1, P_2} q \left(\frac{P_0}{P_1} \right)^\beta \frac{1-F(P_1|\theta_1)}{1-F(\hat{P}_0|\theta_1)} (P_1 - \theta_1 - w_1) + (1-q) \left(\frac{P_0}{P_2} \right)^\beta \frac{1-F(P_2|\theta_2)}{1-F(\hat{P}_0|\theta_2)} (P_2 - \theta_2 - w_2) - q \left[\int_{\hat{P}_0}^{P_1} w_1(P_j) \left(\frac{P_0}{P_j} \right)^\beta d\tilde{F}(P_j|\theta_1) \right] - (1-q) \left[\int_{\hat{P}_0}^{P_2} w_2(P_j) \left(\frac{P_0}{P_j} \right)^\beta d\tilde{F}(P_j|\theta_2) \right] \quad (10)$$

³⁰Alternatively, the principal can infer from the equilibrium exercise policy what type the agent is.

³¹Note that, since $P_j \in (P_j, \bar{P}_j)$, $w_1(P_j) = 0$, and $w_2(P_j) = 0$ elsewhere. Moreover, notice that firm i can never be preempted at a project value greater than the trigger level it follows. Consequently, even though we allow \bar{P}_j to be any value greater than P_1 or P_2 , we can already set $w_1(P_j) = 0$ for $P_j > P_1$ and $w_2(P_j) = 0$ for $P_j > P_2$.

where $\tilde{F}(P|\theta) := \frac{F(P|\theta)}{1-F(\hat{P}_0|\theta)}$.

This optimization is subject to a variety of constraints. The manager is protected by limited-liability and corresponding constraints are,

$$w_1 \geq 0 \tag{11}$$

$$w_2 \geq 0 \tag{12}$$

$$w_1(P_j) \geq 0, \quad w_2(P_j) \geq 0 \quad \forall P_j \in (\underline{P}_j, \overline{P}_j). \tag{13}$$

First of all the participation constraint is,

$$\begin{aligned} & q \left(\frac{P_0}{P_1} \right)^\beta \frac{1 - F(P_1|\theta_1)}{1 - F(\hat{P}_0|\theta_1)} (w_1) + (1 - q) \left(\frac{P_0}{P_2} \right)^\beta \frac{1 - F(P_2|\theta_2)}{1 - F(\hat{P}_0|\theta_2)} (w_2) \tag{14} \\ & + q \left[\int_{\hat{P}_0}^{P_1} w_1(P_j) \left(\frac{P_0}{P_j} \right)^\beta d\tilde{F}(P_j|\theta_1) \right] + (1 - q) \left[\int_{\hat{P}_0}^{P_2} w_2(P_j) \left(\frac{P_0}{P_j} \right)^\beta d\tilde{F}(P_j|\theta_2) \right] \geq 0 \end{aligned}$$

This constraint ensures that the total expected value to the manager of accepting the contract is non-negative.

There exist also constraints due to the hidden information of the manager. These incentive constraints ensure that managers exercise in accordance with the owner's expectations. Particularly, they induce the manager exercise low cost (θ_1) projects at the P_1 trigger and exercise high cost (θ_2) projects at the P_2 trigger. In this setting, the manager with private information have the incentive to lie on the actual project cost and divert free cash flows to themselves. For example, the manager could have an incentive to claim falsely that a lower cost project is a higher cost project and then divert the difference in values³². On the other hand, if profitable enough, the manager could have an incentive to lie and claim that a higher cost project is a lower cost project and then add the difference in values privately to gain eventually a higher wage and payoff. Any incentive compatible contract therefore has to satisfy Eqs. (15) and (16).

³² This could be done by diverting cash for private benefits such as empire building or acquire perquisites.

$$\begin{aligned} & \left(\frac{P_0}{P_1}\right)^\beta \frac{1 - F(P_1|\theta_1)}{1 - F(\hat{P}_0|\theta_1)}(w_1) + \int_{\hat{P}_0}^{P_1} w_1(P_j) \left(\frac{P_0}{P_j}\right)^\beta d\tilde{F}(P_j|\theta_1) \geq \\ & \left(\frac{P_0}{P_2}\right)^\beta \frac{1 - F(P_2|\theta_1)}{1 - F(\hat{P}_0|\theta_1)}(w_2 + \Delta\theta) + \int_{\hat{P}_0}^{P_2} w_2(P_j) \left(\frac{P_0}{P_j}\right)^\beta d\tilde{F}(P_j|\theta_1) \end{aligned} \quad (15)$$

$$\begin{aligned} & \left(\frac{P_0}{P_1}\right)^\beta \frac{1 - F(P_1|\theta_2)}{1 - F(\hat{P}_0|\theta_2)}(w_1 - \Delta\theta) + \int_{\hat{P}_0}^{P_1} w_1(P_j) \left(\frac{P_0}{P_j}\right)^\beta d\tilde{F}(P_j|\theta_2) \leq \\ & \left(\frac{P_0}{P_2}\right)^\beta \frac{1 - F(P_2|\theta_2)}{1 - F(\hat{P}_0|\theta_2)}(w_2) + \int_{\hat{P}_0}^{P_2} w_2(P_j) \left(\frac{P_0}{P_j}\right)^\beta d\tilde{F}(P_j|\theta_2) \end{aligned} \quad (16)$$

The second constraint is shown not to bind, so only constraint Eq. (15) is relevant for our discussion. It ensures that a manager of a low cost project chooses to exercise at P_1 . By truthfully revealing the privately observed cost θ_1 through exercising at P_1 , the manager receives the wage w_1 . By misrepresenting the private cost and waiting until the trigger P_2 , the manager receives the wage w_2 in case she is not preempted, or receives the wage $w_2(P_j)$ in case she is preempted. As a result, Eq. (15) ensures the expected present value of payoff from truthful revelation to be greater than or equal to the expected present value of the payoff from misreporting the private cost. Regarding these constraints, we follow the analysis by Grenadier and Wang (2005). They are common in the literature on hidden information or cash diversion. For example, similar truth-telling conditions appear in Bolton and Scharfstein (1990) and DeMarzo and Sannikov (2006).

All in all, the owner's problem can be summarized as the solution of the objective function in Eq. (10), subject to a total of seven inequality constraints: one participation, two incentive constraints, and four limited-liability constraints.³³ The problem can be substantially simplified in that we can reduce the number of relevant constraints to two, which corresponds to the incentive constraint for a manager of a high cost (θ_2) project and the limited liability constraint for the high cost (θ_2) type.

Analysis

Although the owner's optimization problem is subject to seven inequality constraints, the solution can be found through considering only two of the constraints. Appendix B

³³ Of course there is a continuum of limited liability constraints for the preemption wages.

contains the proves of four propositions: Propositions (8) — (11), that provide the underpinnings for this simplification. Proposition 8 shows that the limited-liability constraint for a manager of a project with θ_1 cost does not bind, while Proposition 9 shows that the participation constraint, Eq. (14), does not bind. Proposition 10 demonstrates that the limited liability constraint for a manager of a high cost (θ_2) type project binds, i.e. $w_2 = 0$. Proposition 11 implies that the incentive constraint for a manager of a high cost (θ_2) type project is never binding.

Solving the reduced program results in an optimal contract specifying exercise trigger/wage/pre-emption wage triples $(P_i, w_i, w_i(P_j))$, $i \in \{1, 2\}$, inducing the agent to reveal the truth and to deliver the corresponding cash flows without diverting to the owner. The precise form of the contract is stated in the following proposition.

Proposition 3 *Under the optimal contract in the setting with preemption threat, the manager of a low cost project exercises the investment option at P_1 receiving a wage of w_1 , given by,*

$$P_1 = -\frac{\beta + \hat{h}(P_1|\theta_1)}{1 - \beta - \hat{h}(P_1|\theta_1)}\theta_1 \quad (17)$$

$$w_1 = \left(\frac{P_1}{P_2}\right)^\beta \frac{1 - F(P_2|\theta_1)}{1 - F(P_1|\theta_1)}\Delta\theta. \quad (18)$$

The manager of a high cost project receives a zero wage, i.e. $w_2 = 0$, and exercises at the trigger,

$$P_2 = -\frac{\beta + \hat{h}(P_2|\theta_2)}{1 - \beta - \hat{h}(P_2|\theta_2)} \left[\theta_2 + \left(\frac{\beta + \hat{h}(P_2|\theta_1)}{\beta + \hat{h}(P_2|\theta_2)} \right) \left(\frac{1 - F(P_2|\theta_1)}{1 - F(P_2|\theta_2)} \right) \frac{q}{1 - q} \Delta\theta \right]. \quad (19)$$

The proof is in Appendix B.

No Correlation

In interpreting the above results, we will highlight the effects of competition and agency on the optimal exercise policy as well as their interactions. To gain a basic intuition let us first consider the case where the firm's own costs (θ) and the competitor's trigger level (P_j) are independent, consequently there exists no correlation between them³⁴. This implies

³⁴ Throughout the analysis, when we use the term uncorrelated we actually mean independence.

$F(P_j|\theta) = F(P_j)$ and $h(P_j|\theta) = h(P_j)$. In that case the trigger levels specified in the optimal contract simplify to the following expressions,

$$P_1 = -\frac{\beta + \hat{h}(P_1)}{1 - \beta - \hat{h}(P_1)}\theta_1 = P_1^{**} \quad (20)$$

$$P_2 = -\frac{\beta + \hat{h}(P_2)}{1 - \beta - \hat{h}(P_2)} \left[\theta_2 + \frac{q}{1-q}\Delta\theta \right] > P_2^{**} \quad (21)$$

$$w_1 = \left(\frac{P_1}{P_2} \right)^\beta \frac{1 - F(P_2)}{1 - F(P_1)} \Delta\theta. \quad (22)$$

Comparing these expressions with the *first-best benchmark* results, given by Eq. (5), we find that, as usual in adverse selection models, there is no distortion at the top (low cost, or equivalently high quality, project), in the sense that the low cost project is exercised at the same level as it would be in a model without agency ($P_1 = P_1^{**}$), whereas the high cost project is exercised later ($P_2 > P_2^{**}$) in order to satisfy the incentive constraint for the manager of a low cost project. Thus, allowing for agency in real investment under competition does not alter the low cost project's exercising trigger but increases the high cost project's exercising trigger. The $\left[\frac{\beta + \hat{h}(P)}{\beta + \hat{h}(P) - 1} \right]$ terms stem from the presence of competition, while the additional term to θ_2 , that is $\left[\frac{q}{1-q}\Delta\theta \right]$, comes from the agency conflicts. The superimposition of the two effects is clearly seen in these expressions above, when compared to the *first-best benchmark* results. Note that the position of P_2 relative to *basic model* results (P_2^*), given by Eq. (3), is ambiguous; while, agency has a tendency to increase high cost project's exercising trigger, competition puts downward pressure on it. The overall effect depends on model parameters, in particular on the severity of the preemption threat relative to the agency conflict. Therefore, both over- and under-investment can occur within the investment decision of a managerial firm under preemption threat. Notice that, under some circumstances competition could fully offset the inefficiency stemming from agency, and bring the high cost project's exercising trigger back to the efficient trigger level³⁵.

Under the optimal contract in the setting with preemption threat, the manager of a low

³⁵ We define the inefficiency as any deviation from the basic model optimal exercise triggers. Because this results in a decrease in the (maximized) surplus that can be created by the real option. If information were complete and/or cooperative behavior were possible among competitors, the efficiency loss would be mitigated. This is similar to the definition of [Lambrecht and Perraudin \(2003\)](#). In the context of agency conflict, our definition is also in line with the one of [Grenadier and Wang \(2005\)](#). They define social loss as the difference between the values of the basic model option value, and the sum of the owner and manager options. They suggest that social loss is driven by the distance of the triggers to the optimal exercise triggers of the basic model.

cost project exercises the investment option at P_1 and receives the wage w_1 . As seen in Eq. (22), w_1 is the product of $\Delta\theta$ and two factors. These factors that determine w_1 are $(\frac{P_1}{P_2})^\beta$ and $[\frac{1-F(P_2)}{1-F(P_1)}]$; while the former represents the discounting effect and it exists even in the absence of competition, the latter represents the effect of competition on agency. First, since $\beta > 1$ and $P_1 < P_2$, the discounting factor decreases with the gap between the triggers. That translates into, the bigger the gap the less wage has to be provided to the manager of a low cost project. Thus, the idea behind the fact that under the optimal contract the high cost project is exercised later, is about making use of the increased gap to provide additional incentives to the manager. Second, $[\frac{1-F(P_2)}{1-F(P_1)}]$ is the ratio of the probability of not being preempted if the manager follows the P_2 exercise trigger, to the probability of not being preempted if the manager follows the P_1 exercise trigger. Because $P_1 < P_2$, this ratio is less than unity and it decreases the wage the manager of low cost project requires. In short, the competition provides additional incentives to the manager for truth-telling and this is intuitive; if the manager of low cost project does not reveal the truth and decides to exercise the project later at P_2 , then there is the possibility of being preempted, and consequently, of neither receiving any wages nor diverting any amount ($\Delta\theta$) for private benefits.

Correlation

Let us consider the more general case where firm i 's own cost (θ) is *positively* correlated with the competitor's trigger level (P_j)³⁶, which could be motivated for instance by the existence of a common component in both firms' costs or a demand shock affecting the industry to which these firms belong (cf. Hart (1983)). Regarding the correlation we explicitly impose *hazard rate dominance*, i.e. $h(P_j|\theta_1) \geq h(P_j) \geq h(P_j|\theta_2)$.³⁷ Introducing correlation has interesting additional impact on the interaction between agency and competition within the real investment framework. Before investigating that, we first analyze the *entrepreneurial* firm under preemption threat. Observing the firm's own costs leads to an update of the belief about the competitor's trigger level. In case of a low cost project, the early exercise of the competitor is more likely. Due to the increased preemption threat, the optimal exercise trigger of firm i is adjusted downwards, i.e. $P_1^{**,corr} < P_1^{**,nocorr}$. On the other hand, in case of a high cost project, the preemption threat is lower. Consequently, the

³⁶ Remember that we simply consider the optimal response of firm i to a given conditional distribution of the competitor's exercising trigger level.

³⁷ Notice again that hazard rate dominance implies first order stochastic dominance, which is often a necessary technical assumption in the adverse selection models.

optimal exercise trigger of firm i is adjusted upwards, i.e. $P_2^{**,corr} > P_2^{**,nocorr}$. Notice that this effect of correlation is present independent of the agency problem and can be seen by analyzing Eq. (5)³⁸, which is a part of the first-best benchmark model results.

Now consider a *managerial* firm under preemption threat. First of all, the mentioned effect of correlation on competition, via the $\frac{\beta+\hat{h}(P|\theta)}{\beta+\hat{h}(P|\theta)-1}$ terms in Eqs. (17) and (19), is also present in this case. Additionally, as seen in Eq. (19), there exists a term multiplying the agency distortion, that is $A := \left[\frac{\beta+\hat{h}(P_2|\theta_1)}{\beta+\hat{h}(P_2|\theta_2)} \right] * \left[\frac{1-F(P_2|\theta_1)}{1-F(P_2|\theta_2)} \right]$. This term represents the impact of correlation on agency. We analyze it within a numerical example in the next subsection and find that when the model parameters are set reasonably, it is less than unity³⁹, and actually even closer to zero than to unity. This means that, the presence of correlation dampens largely the increase in the high cost project's exercising trigger which originally stems from the agency conflict. Next, as seen in Eq. (18), the correlation influences w_1 via the factor that represents the effect of competition on agency, i.e. $\left[\frac{1-F(P_2|\theta_1)}{1-F(P_1|\theta_1)} \right]$. Due to the first order stochastic dominance property of $F(P|\theta)$, positive correlation pushes this term further below unity⁴⁰. Therefore, the gap between the triggers provides additional incentives to the manager for truth-telling. The intuition behind is simple: The manager of low cost project knows that it is more likely that the competitor has lower trigger levels. This decreases greatly the incentives for the low cost project manager to follow untruthfully the P_2 exercise trigger. Because, the probability of being preempted and receiving nothing is actually higher than the one in the absence of correlation. To summarize, the correlation on top of competition provides additional incentives to the manager for truth-telling and it dampens the distortion in the high cost project's exercising trigger⁴¹ which originally stemmed from agency conflict.

A Numerical Example

To illustrate our results and to analyze the $A := \left[\frac{\beta+\hat{h}(P_2|\theta_1)}{\beta+\hat{h}(P_2|\theta_2)} \right] * \left[\frac{1-F(P_2|\theta_1)}{1-F(P_2|\theta_2)} \right]$ term, we present in this subsection a numerical example. We do so for the basic model and for various cases of the model with additional agency conflict and/or preemption fear, with/without the presence of positive correlation. Table (1) shows the exercise triggers for the low cost

³⁸ by particularly noting that $\frac{\partial}{\partial \hat{h}} \left[\frac{\beta+\hat{h}}{\beta+\hat{h}-1} \right] < 0$ and taking the assumption on the hazard rate dominance into account

³⁹ Note that, $A = 1$ when there is no correlation.

⁴⁰ A further assumption, such as $\hat{h}(P|\theta)$ is increasing in P , is needed. We made already such an assumption to satisfy the second-order condition during the proof of the Proposition 2. This is true for standard distributions such as uniform, negative exponential, Weibull and Pareto.

⁴¹ that is $\left[\frac{q}{1-q} \Delta \theta \right]$

(P_1) and high cost (P_2) projects under the optimal contract, as well as the wage (w_1) that the manager of a low cost project receives. Model parameters are set as $\beta = 2$, $\theta_1 = 10$, $\theta_2 = 30$, $q = 0.5$ ⁴². We use the negative exponential distribution for $F(P_j|\theta_i)$, whose c.d.f. is $F(P_j|\theta_i) = 1 - e^{-P_j*[0.5/\theta_i]}$ and adjusted hazard rate is $\hat{h}(P_j|\theta_i) = P_j * [0.5/\theta_i]$, for $i \in \{1, 2\}$. Modeling it this way allows for positive correlation and particularly implies that $E(P_j|\theta_1)$ is set to be equal to P_1^* , and $E(P_j|\theta_2)$ is set to be equal to P_2^* . For the case of no correlation, the unconditional c.d.f is $F(P_j) = 1 - e^{-P_j*[0.5/\bar{\theta}]}$ and adjusted hazard rate is $\hat{h}(P_j) = P_j * [0.5/\bar{\theta}]$, where $\bar{\theta} = (\theta_1 + \theta_2)/2$. That results in the unconditional expectation $E(P_j)$ to be set as $(P_1^* + P_2^*)/2$ and overall we attain the first order stochastic dominance property⁴³. Note that, for the rest of the analysis we use the notation of $F(P_j|.)$ to represent all the conditional, $F(P_j|\theta_i)$, and the unconditional, $F(P_j)$, distributions⁴⁴. As a robustness check we investigate also the cases where we set the parameters: $q = 0.25$, $q = 0.75$ and/or $\beta = 1.5$, $\beta = 5$ as well as where we model $F(P_j|\theta_i)$ as uniform distribution. Our results remain still valid. Notice that, the model parameters are set reasonably and extreme values for the parameters, such as β values very close to 1, q values very close to 0 and 1, are avoided.

The $\frac{P_1}{20.00}$ and $\frac{P_2}{60.00}$ values in Table (1) represent the normalized deviation in the exercise triggers as a multiple of the basic model triggers, that are $P_1^* = 20.00$ and $P_2^* = 60.00$. The $\frac{P_1}{20.00}$ values correspond to the effect of only competition&correlation. However, since the P_2 exercise trigger value can be affected by both the agency problem and the competition&correlation, the $\frac{P_2}{60.00}$ values represent an overall impact. While the $\left[\frac{\beta + \hat{h}(P|.)}{\beta + \hat{h}(P|.) - 1} \right] / \left[\frac{\beta}{\beta - 1} \right]$ term represents the role of competition in this deviation, the $\left[\theta_2 + \frac{q}{1-q} \Delta\theta * A \right] / [\theta_2]$ term represents the role of agency. Remember that, A is the term reflecting the effect of correlation on agency in Eq. (19). Finally, $(\frac{P_1}{P_2})^\beta$ and $\left[\frac{1 - F(P_2|.)}{1 - F(P_1|.)} \right]$ are the coefficients of $\Delta\theta$ in Eq. (18); while the former represents the discounting effect, the latter represents the effect of competition on agency.

First, comparing the results of the model with only agency conflict (column 2) to the basic model results, we see that there is no impact of the agency on P_1 , while P_2 increased to 1.67 times of its basic model value. In the case where there is additionally the preemption fear

⁴² To achieve $\beta = 2$, one can set $\alpha = 0.02$, $r = 0.04$, $\sigma = 0.2$

⁴³ For instance, conditional on $\theta = \theta_1$, the $F(P_j|\theta_1)$ is equal to $1 - e^{-P_j*[0.5/10]}$, consequently $E(P_j|\theta_1) = 20$. This means that the manager of a low cost project believes that on average the competitor's trigger level is equal to firm i 's basic model low cost project trigger, which is $P_1^* = 20$.

⁴⁴ The same representation also applies to the adjusted hazard rate notation, $\hat{h}(P_j|.)$.

(column 5), we observe that competition has an effect of lowering both P_1 and P_2 ; P_1 to %85 of its initial value and P_2 to %69 of its initial value. The impact of agency conflict, that is only on P_2 , remained the same and together with the effect of competition P_2 increased to 1.14 times of its basic model value. This illustrates that both over- and under-investment can occur within the investment decision of a managerial firm under preemption threat. Regarding w_1 , we see that compared to the model with only agency conflict (column 2), here $w_1 = 0.34$, which is much smaller than 0.80. That stems from the change in the $\left[\frac{1-F(P_2|\cdot)}{1-F(P_1|\cdot)}\right]$ component of w_1 , which proves that the competition provides additional incentives to the manager for truth-telling. Second, notice the impact of correlation on the exercise triggers, for the model with preemption fear (column 4 relative to column 3): 17.02 slightly decreased to 15.62, while 44.24 slightly increased to 46.85. Next, we examine the effect of positive correlation in the case of *managerial* firm under preemption threat (column 6 relative to column 5). We observe that, the correlation dampens the distortion in the high cost project's exercising trigger which originally stemmed from agency conflict. The coefficient for this distortion, i.e. $\left[\theta_2 + \frac{q}{1-q}\Delta\theta * A\right] / [\theta_2]$, decreased from 1.67 to 1.18, because its component A dropped from 1.00 to 0.28. Regarding w_1 , we see that compared to the model with the absence of correlation, here $w_1 = 0.25$, which is smaller than 0.34. This stems from the $\left[\frac{1-F(P_2|\cdot)}{1-F(P_1|\cdot)}\right]$ part of w_1 , which proves that the correlation on top of competition provides additional incentives to the manager for truth-telling.

3.4 Extensions

In this section, we extend our previous analysis of the principal-agent model (no correlation case) into an equilibrium analysis, as done also in Section 3 of [Lambrecht and Perraudin \(2003\)](#). The aim is to check the robustness of our findings, particularly for the optimal exercise triggers P_1 and P_2 . Before proceeding with the equilibrium analysis, we need to extend our model to allow for continuous distribution of the project costs.

In our core principal-agent model we were restricted to a binary distribution of the unobserved project costs. A natural and straightforward extension is allowing for a continuous distribution of project costs, $G(\theta)$, for $\theta \in [\theta_1, \theta_2]$. In this setting the principal designs a truth-telling contract specifying a function of recommended exercise strategies, P_θ , and a function of corresponding exercise wages, w_θ . Limited-liability requires the wage function to stay non-negative and there exist a continuum of incentive constraints, one for each project cost level. For the solution of the agency problem in this setting, we follow the

Table 1: Exercise triggers for the low cost (P_1) and high cost (P_2) projects under the optimal contract. And, the wage (w_1) that the manager of a low cost project receives (Note that $w_2 = 0$). $\frac{P_1}{20.00}$ and $\frac{P_2}{60.00}$ represent the deviation in the exercise triggers as a multiple of the basic model triggers. $\left[\frac{\beta + \hat{h}(P_j|\cdot)}{\beta + \hat{h}(P_j|\cdot) - 1} \right] / \left[\frac{\beta}{\beta - 1} \right]$ represents the role of competition in this deviation. $\left[\theta_2 + \frac{q}{1-q} \Delta\theta * A \right] / [\theta_2]$ represents the role of agency in this deviation. A is the term reflecting the effect of correlation on agency in Eq. (19). Finally, $\left(\frac{P_1}{P_2} \right)^\beta$ and $\left[\frac{1-F(P_2|\cdot)}{1-F(P_1|\cdot)} \right]$ are the coefficients of $\Delta\theta$ in Eq. (18); while the former represents the discounting effect, the latter represents the effect of competition on agency. Model parameters are set as $\beta = 2$, $\theta_1 = 10$, $\theta_2 = 30$, $q = 0.5$. For the negative exponential distribution, the c.d.f. is $F(P_j|\cdot) = 1 - e^{-P_j * [0.5/\cdot]}$ and the adjusted hazard rate is $\hat{h}(P_j|\cdot) = P_j * [0.5/\cdot]$.

	<i>Basic</i>	<i>Agen.</i>	<i>Comp.</i>	<i>Comp.</i> <i>+Corr.</i>	<i>Agen.</i> <i>+Comp.</i>	<i>Agen.</i> <i>+Comp.</i> <i>+Corr.</i>
P_1	20.00	20.00	17.02	15.62	17.02	15.62
$\frac{P_1}{20.00} = \left[\frac{\beta + \hat{h}(P_1 \cdot)}{\beta + \hat{h}(P_1 \cdot) - 1} \right] / \left[\frac{\beta}{\beta - 1} \right]$	1.00	1.00	0.85	0.78	0.85	0.78
P_2	60.00	100.00	44.24	46.85	68.44	53.98
$\frac{P_2}{60.00}$	1.00	1.67	0.74	0.78	1.14	0.90
$\left[\frac{\beta + \hat{h}(P_2 \cdot)}{\beta + \hat{h}(P_2 \cdot) - 1} \right] / \left[\frac{\beta}{\beta - 1} \right]$	1.00	1.00	0.74	0.78	0.69	0.77
$\left[\theta_2 + \frac{q}{1-q} \Delta\theta * A \right] / \theta_2$	1.00	1.67	1.00	1.00	1.67	1.18
$A := \left[\frac{\beta + \hat{h}(P_2 \theta_1)}{\beta + \hat{h}(P_2 \theta_2)} \right] * \left[\frac{1-F(P_2 \theta_1)}{1-F(P_2 \theta_2)} \right]$	1.00	1.00	1.00	1.00	1.00	0.28
w_1	-	0.80	-	-	0.34	0.25
$\left(\frac{P_1}{P_2} \right)^\beta$	-	0.04	-	-	0.06	0.08
$\left[\frac{1-F(P_2 \cdot)}{1-F(P_1 \cdot)} \right]$	-	1.00	-	-	0.28	0.15

derivations in Appendix A.3. of Grenadier and Wang (2005). This delivers the usual result that the manager of the highest cost project (θ_2) does not receive any rents, and the lowest cost project (θ_1) is exercised at the first-best trigger level. The exercise triggers and the corresponding wages are given in the following proposition,

Proposition 4 *Under the optimal contract in the setting with preemption threat (no correlation case), the manager of a project with cost equal to $\theta \in [\theta_1, \theta_2]$, exercises the investment option at P_θ and receives a wage of w_θ , given by,*

$$P_\theta = -\frac{\beta + \hat{h}(P_\theta)}{1 - \beta - \hat{h}(P_\theta)} \left(\theta + \frac{G(\theta)}{G'(\theta)} \right) \quad (23)$$

$$w_\theta = \int_{\theta}^{\theta_2} \left(\frac{P_\theta}{P(s)} \right)^\beta ds. \quad (24)$$

The proof is in Appendix B.

This extension is particularly valuable for analyzing the symmetric (Bayesian Nash) equilibrium, as described in the following paragraphs.

So far, our attention has been restricted to the behavior of a managerial firm facing preemption risk. Our analysis is based on deriving the optimal response of a firm, given the distribution of the competitor's optimal exercising trigger level. One could step back and start with the competitor's project cost (or quality) distribution and determine the (Bayesian Nash) equilibrium⁴⁵ trigger level distribution endogenously, in which both firms take strategic real investment decisions. We follow the analysis in Section 3 of Lambrecht and Perraudin (2003)⁴⁶ and derive the symmetric (Bayesian Nash) equilibrium exercise trigger values as follows.

Assume that there are two firms, labeled $i = 1, 2$, each of which can invest in the project, $P(t)$, for a cost, θ^i . When one firm invests, however, the opportunity to invest is lost to the other firm. If the two firms invest simultaneously, with probability 1/2, firm i ($i = 1, 2$) receives the project value at a cost θ^i while firm j ($j \neq i$) gets nothing. We

⁴⁵ For an introduction to the theory of static games of incomplete information and Bayesian Nash equilibrium, cf. for example Ch.6 of Fudenberg and Tirole (1993); in particular the discussion of first-price auctions with continuum of types is valuable to the analysis here.

⁴⁶In this study, there is complete analysis of Bayesian Nash equilibrium in the context of real option exercise games with incomplete information and under preemption threat, but without the presence of agency conflicts.

introduce incomplete information by supposing that the i th firm observes its own cost, θ^i , but knows only that θ^j , $j \neq i$, is an independent draw from a distribution $G(\theta)$. $G(\theta)$ has a continuously differentiable density, $G'(\theta)$, with strictly positive support on an interval $[\theta_1, \theta_2]$.

Returning to the derivation of the model, we note that, in equilibrium, each firm's investment trigger $P^{eqb,i}$ may be regarded as the level of a mapping $P^{eqb,i}(\theta)$, $i = 1, 2$, from its cost parameter, θ , to its optimal strategic investment trigger, $P_\theta^{eqb,i}$. Before solving for the equilibrium mappings, we demonstrate some important properties that the mappings possess. In doing this, we need to impose a regularity condition on the distribution function, $G(\theta)$, such that for $\theta \in [\theta_1, \theta_2]$, $\theta G'(\theta)/(1 - G(\theta))$ is increasing in θ . As noted previously most standard distributions possess this property.

Proposition 5 *Under the assumptions above, for each firm, $i=1, 2$, the mapping, $P^{eqb,i}(\theta)$, from the investment cost, θ , to the optimal investment trigger, $P_\theta^{eqb,i}$, is strictly increasing. If the investment cost distribution, $G(\theta)$, satisfies the regularity condition mentioned above, then $P^{eqb,i}(\theta)$ is continuous. Finally, for $i=1, 2$ the values of the $P^{eqb,i}(\theta)$ functions coincide at the upper and lower ends of the support of the θ distribution, i.e., $P^{eqb,1}(\theta_1)=P^{eqb,2}(\theta_1)$ and $P^{eqb,1}(\theta_2)=P^{eqb,2}(\theta_2)$.*

The proof is in Appendix B.

If the cost parameter and investment trigger are linked by a continuous, strictly increasing function $P^{eqb,i}(\theta)$, the rational conjecture for firm i ($i = 1, 2$) to adopt is that the distribution of firm j 's ($j \neq i$) investment trigger is,

$$F^j(P^{eqb}) = G(\theta^j(P^{eqb})), \quad (25)$$

where $\theta^j(P^{eqb})$ is the inverse of $P^{eqb,j}(\theta)$, and the support of the F^j distribution is $[P_1, P_2]$, where $P_1 = P^{eqb,i}(\theta_1)$ and $P_2 = P^{eqb,i}(\theta_2)$ for $i = 1, 2$.

Rearranging the Eq. (23) and using the fact that,

$$\hat{h}^j(P) = \frac{PF^{j'}(P)}{1 - F^j(P)} = \frac{PG'(\theta)}{1 - G(\theta)}\theta^{j'}(P), \quad (26)$$

we obtain the following system of non-linear differential equations for the two functions, $\theta^i(P)$, $i = 1, 2$.

$$\theta^{1'}(P) = \frac{1 - G(\theta^1(P))}{G'(\theta^1(P))} \left(\frac{1}{P - (\theta^2 + \frac{G(\theta^2(P))}{G'(\theta^2(P))})} - \frac{\beta}{P} \right) \quad (27)$$

$$\theta^{2'}(P) = \frac{1 - G(\theta^2(P))}{G'(\theta^2(P))} \left(\frac{1}{P - (\theta^1 + \frac{G(\theta^1(P))}{G'(\theta^1(P))})} - \frac{\beta}{P} \right) \quad (28)$$

Regarding the boundary condition, if $P(t) \rightarrow P_2$ but neither firm has so far invested, then each firm knows that the other will preempt almost certainly in the next few instants. Consequently, the hazard of being preempted per unit of time explodes to infinity. Thus, the relevant boundary conditions are: $\theta^i(\theta_2 + \frac{1}{G'(\theta_2)}) = \theta_2$ for $i = 1, 2$.

The system of differential equations in (27) and (28) allow us to deduce the following result:

Proposition 6 *Under the assumptions of this section, there is a unique symmetric equilibrium in which each firm's optimal investment trigger is the solution to the differential equation:*

$$\theta'(P^{eqb}) = \frac{1 - G(\theta)}{G'(\theta)} \left(\frac{1}{P^{eqb} - (\theta + \frac{G(\theta)}{G'(\theta)})} - \frac{\beta}{P^{eqb}} \right) \quad (29)$$

subject to $\theta(\theta_2 + \frac{1}{G'(\theta_2)}) = \theta_2$.

The proof is in Appendix B.

Now, we describe a simple distribution that yields convenient closed-form or numerically easy to solve solutions. Suppose that the investment cost distribution, $G(\theta)$, is isoelastic with a bounded support, i.e.,

$$G(\theta) = \frac{\theta_1^{-\alpha} - \theta^{-\alpha}}{\theta_1^{-\alpha} - \theta_2^{-\alpha}} \quad (30)$$

for $\theta \in [\theta_1, \theta_2]$, where $0 < \theta_1 < \theta_2 < \infty$, and $\alpha \neq 0$

This distribution is commonly referred to as the Pareto distribution. For this distribution, the function $P^{eqb}(\theta)$ satisfies,

$$P^{eqb'}(\theta) = \frac{\alpha P^{eqb} \theta^{-\alpha-1}}{\theta^{-\alpha} - \theta_2^{-\alpha}} \left(\frac{P^{eqb} - (\theta + \frac{\theta_1^{-\alpha} - \theta^{-\alpha}}{\alpha \theta^{-\alpha-1}})}{P^{eqb} - \beta(P^{eqb} - (\theta + \frac{\theta_1^{-\alpha} - \theta^{-\alpha}}{\alpha \theta^{-\alpha-1}}))} \right) \quad (31)$$

subject to $P^{eqb}(\theta_2) = \theta_2 + \frac{\theta_1^{-\alpha} - \theta_2^{-\alpha}}{\alpha\theta_2^{-\alpha-1}}$.

This equation may be solved numerically with little difficulty. Alternatively, a simple closed-form solution may be obtained by driving θ_2 , the upper bound of the support of $G(\theta)$, to infinity.

Proposition 7 *If $G(\theta)$ satisfies Eq. (30) and $\alpha > 0$, as θ_2 goes to infinity, the optimal investment trigger P_θ^{eqb} converges to a constant proportion of the investment cost distorted due to the agency conflict:*

$$P_\theta^{eqb} = -\frac{\beta + \alpha}{1 - \beta - \alpha} \left(\theta + \frac{\theta_1^{-\alpha} - \theta^{-\alpha}}{\alpha\theta^{-\alpha-1}} \right) \quad (32)$$

The proof is in Appendix B.

Let us examine the equilibrium results for P_1^{eqb} and P_2^{eqb} corresponding to θ_1 and θ_2 respectively.

Regarding P_2^{eqb} , we will make use of the boundary condition itself,

$$P_{\theta_2}^{eqb} = \theta_2 + \frac{\theta_1^{-\alpha} - \theta_2^{-\alpha}}{\alpha\theta_2^{-\alpha-1}}. \quad (33)$$

We observe that, due to strong competition the trigger value tends to reduce to the NPV trigger level, θ_2 . Meanwhile, the agency calls for a higher trigger value, and the overall effect is dependent on the relative importance of preemption threat to the agency conflict. Notice that, in equilibrium, when $P(t)$ is close to the $P_{\theta_2}^{eqb}$ trigger level, the competition is stronger than the one in the previous section⁴⁷, when $P(t)$ was close to the P_2 level. This is mainly because in equilibrium, both firms have the same $P_{\theta_2}^{eqb}$ trigger levels, while previously the firm with preemption fear was under a milder threat because the competitor's trigger value had a distribution with an upper end of the support, \bar{P}_j greater than P_2 ⁴⁸. These results are in line with our previous section's results. Remember that the agency distortion on the P_2 trigger level stemmed from the need of the owner to give the manager with the low cost project the truth-telling incentives not to lie and claim that he had a high cost project. For the case of a continuous distribution of project costs, which is utilized for our equilibrium analysis, the owner needs to also give the other possible manager types⁴⁹, the truth-telling

⁴⁷for the investment decision of a *managerial* firm under fear of preemption

⁴⁸the optimal trigger level for a *managerial* firm under fear of preemption and owning a high cost (θ_2) project

⁴⁹with project costs between the highest and the lowest costs

incentives not to lie as a higher cost project owner. Consequently, in equilibrium we expect the agency distortion on $P^{eqb}(\theta_2)$ to be greater than it was previously.

Regarding P_1^{eqb} , we will make use of Eq. (32), which we obtained when $\alpha > 0$ and as θ_2 goes to infinity. If we plug in $\theta = \theta_1$ to this equation we obtain,

$$P_{\theta_1}^{eqb} = -\frac{\beta + \alpha}{1 - \beta - \alpha}\theta_1 \quad (34)$$

We observe that in equilibrium, there is no efficiency distortion due to agency at the low cost project trigger level, but only due to competition. Moreover, the equilibrium trigger level for the low cost project in Eq. (34) is equivalent to the one in Eq. (20), P_1 , for the *managerial* firm under fear of preemption and owning a low cost (θ_1) project. Because, for the Pareto distribution (when $\alpha > 0$ and as θ_2 goes to infinity) the adjusted hazard rate, $\hat{h}(P)$, is equal to α . Thus, we can conclude that the results for the low cost project trigger are also in line with our previous section's findings.

3.5 Model Limitations

In this section we discuss the limitations of our principal-agent model described in the previous sections. First, in our way of modeling the owner and the manager value payoffs indifferently. One might want to relax the assumption that both the principal and the agent are equally patient. Impatience can be modeled for agents by assigning them a discount factor different (greater) than β , that is used to discount future cash flows. Particularly, managerial impatience impact the incentive constraints. Thus, this generalization can alter the predictions about investment timing. Grenadier and Wang (2005) implement this in their setting and find that on one hand, introducing impatience creates further incentives for the low cost type agent, therefore less distortion at the high cost trigger level is required. On the other hand, they show that there occurs distortion at the low cost trigger level, so that also over-investment can exist in their model. They note that this generalized problem does not change the basis of the problem and much of the solution methodology is the same. Therefore, allowing impatience in our principal-agent model would not alter our core results, but would add one more layer to analyze. It may prove interesting to investigate it.

Second, notice that, the crucial ingredient in our analysis, is incomplete information about the competitor's costs. Under complete information the option value to wait could be

completely destroyed, given the firms have the same costs and preemption is full. Studies analyzing real option exercise games under complete information include for instance Grenadier (1996) , Trigeorgis (1996), Botteron, Chesney, and Gibson-Asner (2003) and Pawlina and Kort (2006). The main finding of this strand of literature is that if a firm is fearful that a competitor may enter the market first, and if further the market is perceived to be not deep enough to support more than one firm, then the option value of delaying may not be very high, and can even become zero.

Next, similar to Lambrecht and Perraudin (2003), we model competition within a full preemption setting, where there is no benefit of investing for the second mover. Augmenting our principal-agent model setting to allow for partial preemption can be done as follows: For each cost realization the principal specifies two trigger/wage pairs; one for the case where the firm moves first, other for moving second. The first mover trigger is expected to be slightly higher as being preempted does not necessarily result in zero profit anymore. The second mover virtually faces no competition any more and hence can invest at the optimal non-strategic (without competition) trigger. Consequently, milder outcomes would be obtained if partial preemption was allowed for. However, since our main results are more emphasized in the full preemption setting, and for simplicity, we restrict our analysis to this case. For partial preemption argument within real options exercise games cf. Botteron, Chesney, and Gibson-Asner (2003) and Pawlina and Kort (2006).

Finally, it is important to note that, there exists a strand of literature on signal extraction, which includes the work by Cremer and McLean (1988). In this paper, the authors consider auctions for a single indivisible object, in the case where the bidders have information about each other which is not available to the seller. They show that the seller can use this information to his own benefit, and they completely characterize the environments in which a well chosen auction gives him the same expected payoff as that obtainable were he able to sell the object with full information about each bidder's willingness to pay. In our model, the manager of a firm has private information about his own costs, as well as has a signal about the other firm's costs, both of which are not available to the owner of the first firm. This signal is due to the existence of a positive correlation between the firms' costs. The analogy between our model and the work by Cremer and McLean (1988) questions the possibility whether the owners in our model can design a more sophisticated wage contract to achieve a similar, less inefficient, outcome. First of all, in Cremer and McLean (1988) there is a single principal. In our model the owners, to

be able to achieve a similar outcome, would need to cooperate, but not compete. More importantly, they need to incorporate the competitor's actions, such as their preemption, into the wage scheme of their managers. This could be in the form of punishing the manager (e.g. offering a negative wage) when he is preempted. But, then this violates the limited liability constraints. Or, maybe offering some positive preemption wages to the manager can help in this direction. However, with our modeling approach and due to the optimality conditions they are found to be zero. Therefore, there will be no contract designed better that will allow the owner to benefit from the privately known information on the competitor, due to the positive correlation between the firms' costs.

4 Conclusion

This paper adds agency to a standard real option exercise game under incomplete information. Competition is modeled as full preemption in the sense that the first mover has the advantage to seize the market fully and the second mover has no value to invest anymore. The delegation of investment decision to a manager creates an agency conflict since investment cost (or project quality) is unobservable to the principal which gives the manager opportunity for diverting part of the cash flows for own private benefits. Thus, an optimal contract has to be designed which induces the agent to reveal truthfully the project's actual costs (or project quality) and exercise the investment option at a strategically optimal trigger level.

Our main results indicate that competition lets the gap between the triggers serve to provide additional incentives to the manager for truth-telling and as a result allows the owner to supply less (informational) rents to the manager. Our other major findings derive from analyzing the effect of positive *correlation* between the competing firms' underlying project values on competition and agency conflict. Introducing correlation has two interesting consequences within our extended real investment framework. First, regarding an *entrepreneurial* firm: While the amount of investment timing adjustment required due to competition decreases for the high cost (or low quality) project, the same increases for the low cost (or high quality) project. Second, regarding a *managerial* firm: The correlation emphasizes the mentioned role of the gap between the triggers for providing additional incentives to the manager for truth-telling⁵⁰. Moreover, the presence of correlation sup-

⁵⁰and as a result, for allowing the owner to provide less (informational) rents to the manager

presses the distortion in the high cost (or low quality) project's exercising trigger that originally stems from the agency problem.

Some extensions of the model would prove interesting. First, stock price reactions according to observed investment behavior of the firm are worth studying. The effect of investment decision announcement (early, on time, delayed) signals private information (on project costs, or equivalently on project quality) to the market. Moreover, this action (or no-action) might contain information about the agency conflicts within the firm, as well as the beliefs of the firm about the competitors. Notice that, since under the efficient-market hypothesis stock price changes reflect the information revelation instantly, the manager's compensation contract can be contingent on the firm's stock price as well. Second, it is promising to investigate the consequences of introducing competition, as well as positive *correlation*, to a model of moral hazard as in Grenadier and Wang (2005), in which the agent can influence the probability of a low cost (high quality) project by exerting costly and unobservable effort.

5 Appendix

Appendix A. Derivations for the Basic and First-Best Benchmark Models

Proof of Proposition 1. The derivation is standard. See for example, Dixit and Pindyck (1994).□

Proof of Proposition 2.

$$V(P_t, \hat{P}_t, \theta; P) = \mathbb{E}_t \left[e^{-r(T_P - t)} (P - \theta) \mathbf{1}_{P_j(\theta) > P} \middle| P_j(\theta) > \hat{P}_t \right] \quad (35)$$

where $T_P = \inf\{t \geq 0; P_t \geq P\}$. P is a predetermined arbitrary exercise trigger and

independent of $P_j(\theta)$, therefore $P_j(\theta)$ and T_P are independent,

$$V(P_t, \hat{P}_t, \theta; P) = \mathbb{E}_t [e^{-r(T_P-t)}(P - \theta)] \mathbb{E}_t [\mathbf{1}_{P_j(\theta) > P} | P_j(\theta) > \hat{P}_t] \quad (36)$$

$$= (P - \theta) \mathbb{E}_t [e^{-r(T_P-t)}] \text{Prob}_t [P_j(\theta) > P | P_j(\theta) > \hat{P}_t] \quad (37)$$

$$= (P - \theta) \left(\frac{P_t}{P} \right)^\beta \text{Prob}_t [P_j(\theta) > P | P_j(\theta) > \hat{P}_t] \quad (38)$$

$$= (P - \theta) \left(\frac{P_t}{P} \right)^\beta \frac{\text{Prob}_t(P_j(\theta) > P)}{\text{Prob}_t(P_j(\theta) > \hat{P}_t)} \quad (39)$$

$$= (P - \theta) \left(\frac{P_t}{P} \right)^\beta \frac{1 - F(P|\theta)}{1 - F(\hat{P}_t|\theta)} \quad (40)$$

$$V(P_0, \hat{P}_0, \theta; P) = (P - \theta) \left(\frac{P_0}{P} \right)^\beta \frac{1 - F(P|\theta)}{1 - F(\hat{P}_0|\theta)}. \quad (41)$$

First order condition with respect to P , $\partial V(P_0, \hat{P}_0, \theta; P)/\partial P = 0$, gives the optimal exercise trigger level,

$$P^{**} = \frac{\beta + \hat{h}(P^{**}|\theta)}{\beta + \hat{h}(P^{**}|\theta) - 1} \theta \quad (42)$$

where $\hat{h}(P|\theta) := \frac{P \cdot F'(P|\theta)}{1 - F(P|\theta)}$. Note that a sufficient although not necessary condition for the second-order condition, $\partial^2 W(P_0, \hat{P}_0, \theta; P)/\partial P^2 < 0$, to hold is that $\hat{h}(P|\theta)$ is increasing in P . This is true for standard distributions such as uniform, negative exponential, Weibull and Pareto⁵¹. \square

Appendix B. Solution to the optimal contracting under the Principal-Agent setting

For simplicity of our analysis and without loss of generality we set $\hat{P}_0 = P_0$; together with our previous assumption $F(P_0|\theta) = 0$ it implies $F(\hat{P}_0|\theta) = 0$ and $\tilde{F}(P_j|\theta) = F(P_j|\theta)$. In order to avoid singularities, we further assume $F(P_2|\theta) \leq 1$ and $F(P_1|\theta) \geq 0$, i.e. the probability that firm j has a trigger level less than or equal to P_1 or greater than or equal to P_2 , is not zero.

Proposition 8 *The limited-liability constraint for a manager of a project with θ_1 cost does not bind, i.e. either $w_1 > 0$ and/or $w_1(P_j) > 0$.*

⁵¹ This setting and the related derivations are covered by Section 2 of [Lambrecht and Perraudin \(2003\)](#).

Proof. Eq. (15) reads as,

$$\begin{aligned} & \left(\frac{P_0}{P_1}\right)^\beta \frac{1 - F(P_1|\theta_1)}{1 - F(\hat{P}_0|\theta_1)}(w_1) + \int_{\hat{P}_0}^{P_1} w_1(P_j) \left(\frac{P_0}{P_j}\right)^\beta d\tilde{F}(P_j|\theta_1) \geq \\ & \left(\frac{P_0}{P_2}\right)^\beta \frac{1 - F(P_2|\theta_1)}{1 - F(\hat{P}_0|\theta_1)}(w_2 + \Delta\theta) + \int_{\hat{P}_0}^{P_2} w_2(P_j) \left(\frac{P_0}{P_j}\right)^\beta d\tilde{F}(P_j|\theta_1) \end{aligned} \quad (43)$$

Together with the limited-liability constraints $w_2 \geq 0$ and $w_2(P_j) \geq 0$, we can simplify the right hand-side of the inequality as,

$$\left(\frac{P_0}{P_1}\right)^\beta \frac{1 - F(P_1|\theta_1)}{1 - F(\hat{P}_0|\theta_1)}(w_1) + \int_{\hat{P}_0}^{P_1} w_1(P_j) \left(\frac{P_0}{P_j}\right)^\beta d\tilde{F}(P_j|\theta_1) \geq \left(\frac{P_0}{P_2}\right)^\beta \frac{1 - F(P_2|\theta_1)}{1 - F(\hat{P}_0|\theta_1)}(\Delta\theta) \quad (44)$$

Since, $\Delta\theta > 0$ and $\left(\frac{P_0}{P_2}\right)^\beta \frac{1 - F(P_2|\theta_1)}{1 - F(\hat{P}_0|\theta_1)}$ is non-negative, we can write,

$$\left(\frac{P_0}{P_1}\right)^\beta \frac{1 - F(P_1|\theta_1)}{1 - F(\hat{P}_0|\theta_1)}(w_1) + \int_{\hat{P}_0}^{P_1} w_1(P_j) \left(\frac{P_0}{P_j}\right)^\beta d\tilde{F}(P_j|\theta_1) > 0 \quad (45)$$

The terms in Eq. (45), $\left(\frac{P_0}{P_1}\right)^\beta \frac{1 - F(P_1|\theta_1)}{1 - F(\hat{P}_0|\theta_1)}$ and $\left(\frac{P_0}{P_j}\right)^\beta$ are both non-negative. That implies that either either $w_1 > 0$ and/or $w_1(P_j) > 0$.

□

Proposition 9 *The participation constraint, Eq. (14), does not bind.*

Proof. First, we multiply Eq. (45) with $q > 0$ on both sides,

$$q \left(\frac{P_0}{P_1}\right)^\beta \frac{1 - F(P_1|\theta_1)}{1 - F(\hat{P}_0|\theta_1)}(w_1) + q \int_{\hat{P}_0}^{P_1} w_1(P_j) \left(\frac{P_0}{P_j}\right)^\beta d\tilde{F}(P_j|\theta_1) > 0 \quad (46)$$

Next, due to the limited-liability constraints $w_2 \geq 0$ and $w_2(P_j) \geq 0$, and because all the remaining terms below are also non-negative, we can write,

$$(1 - q) \left(\frac{P_0}{P_2} \right)^\beta \frac{1 - F(P_2|\theta_2)}{1 - F(\hat{P}_0|\theta_2)}(w_2) + (1 - q) \int_{\hat{P}_0}^{P_2} w_2(P_j) \left(\frac{P_0}{P_j} \right)^\beta d\tilde{F}(P_j|\theta_2) \geq 0 \quad (47)$$

The sum of two expressions above results in the participation constraint not binding,

$$\begin{aligned} & q \left(\frac{P_0}{P_1} \right)^\beta \frac{1 - F(P_1|\theta_1)}{1 - F(\hat{P}_0|\theta_1)}(w_1) + (1 - q) \left(\frac{P_0}{P_2} \right)^\beta \frac{1 - F(P_2|\theta_2)}{1 - F(\hat{P}_0|\theta_2)}(w_2) \\ & + q \int_{\hat{P}_0}^{P_1} w_1(P_j) \left(\frac{P_0}{P_j} \right)^\beta d\tilde{F}(P_j|\theta_1) + (1 - q) \int_{\hat{P}_0}^{P_2} w_2(P_j) \left(\frac{P_0}{P_j} \right)^\beta d\tilde{F}(P_j|\theta_2) > 0. \end{aligned}$$

□

Propositions 8 and 9 allow us to express the owner's maximization problem as the objective in Eq. (10), subject to Eqs. (12), (13), (15) and (16). The Lagrangian reads,

$$\begin{aligned}
\mathcal{L} = & \left[\left(\frac{P_0}{P_1} \right)^\beta \frac{1 - F(P_1|\theta_1)}{1 - F(P_0|\theta_1)} (P_1 - \theta_1 - w_1) - \int_{P_0}^{P_1} w_1(P_j) \left(\frac{P_0}{P_j} \right)^\beta dF(P_j|\theta_1) \right] (48) \\
& + \left(\frac{1-q}{q} \right) \left[\left(\frac{P_0}{P_2} \right)^\beta \frac{1 - F(P_2|\theta_2)}{1 - F(P_0|\theta_2)} (P_2 - \theta_2 - w_2) - \int_{P_0}^{P_2} w_2(P_j) \left(\frac{P_0}{P_j} \right)^\beta dF(P_j|\theta_2) \right] \\
& + \lambda_1 \left[\left(\frac{P_0}{P_1} \right)^\beta \frac{1 - F(P_1|\theta_1)}{1 - F(P_0|\theta_1)} (w_1) - \left(\frac{P_0}{P_2} \right)^\beta \frac{1 - F(P_2|\theta_1)}{1 - F(P_0|\theta_1)} (w_2 + \Delta\theta) \right. \\
& \quad \left. + \int_{P_0}^{P_1} w_1(P_j) \left(\frac{P_0}{P_j} \right)^\beta d\tilde{F}(P_j|\theta_1) - \int_{P_0}^{P_2} w_2(P_j) \left(\frac{P_0}{P_j} \right)^\beta dF(P_j|\theta_1) \right] \\
& + \lambda_2 \left[\left(\frac{P_0}{P_2} \right)^\beta \frac{1 - F(P_2|\theta_2)}{1 - F(P_0|\theta_2)} (w_2) + \int_{P_0}^{P_2} w_2(P_j) \left(\frac{P_0}{P_j} \right)^\beta dF(P_j|\theta_2) \right. \\
& \quad \left. - \left(\frac{P_0}{P_1} \right)^\beta \frac{1 - F(P_1|\theta_2)}{1 - F(P_0|\theta_2)} (w_1 - \Delta\theta) - \int_{P_0}^{P_1} w_1(P_j) \left(\frac{P_0}{P_j} \right)^\beta d\tilde{F}(P_j|\theta_2) \right] \\
& + \lambda_3 w_2 \\
& + \int_{P_0}^{P_1} \lambda_4(j) w_1(P_j) dP_j \\
& + \int_{P_0}^{P_2} \lambda_5(j) w_2(P_j) dP_j
\end{aligned}$$

with corresponding complementary slackness conditions for the five constraints.

At this point, remember that $P_j \in (\underline{P}_j, \overline{P}_j)$ and we showed previously that $w_1(P_j) = 0$ for $P_j > P_1$, and $w_2(P_j) = 0$ for $P_j > P_2$. Now, let us show that $w_2(P_j) = 0$ is valid also for $\forall P_j \in (\underline{P}_j, P_2)$. First, notice that any positive value of $w_2(P_j)$, compared to the zero value, decreases the objective function of the principal. Second, we know that the incentive constraint Eq. (15) binds, but the incentive constraint Eq. (16) does not bind. Moreover, for any positive value of $w_2(P_j)$, Eq. (15) gets stricter; i.e. for any adjustment made on the variables in order to satisfy Eq. (15) back, the objective function would get worse. On the other hand, for any positive value of $w_2(P_j)$, Eq. (16) gets looser; i.e. the not binding constraint remains not binding, therefore it does not adversely influence the objective function. In summary, any positive value of $w_2(P_j)$ can only worsen the objective function while feasibility is kept within the two incentive constraints. Overall, we can conclude that $w_2(P_j) = 0$ for $\forall P_j \in (\underline{P}_j, \overline{P}_j)$.

Regarding the $w_1(P_j)$, for $\forall P_j \in (\underline{P}_j, P_1)$, let us point to the term below,

$$\left[\left(\frac{P_0}{P_1} \right)^\beta \frac{1-F(P_1|\theta)}{1-F(P_0|\theta)} (w_1) + \int_{P_0}^{P_1} w_1(P_j) \left(\frac{P_0}{P_j} \right)^\beta dF(P_j|\theta) \right].$$

Notice that, w_1 and $w_1(P_j)$ appear in the objective function and in all of the relevant constraints, always together and within the form of the term above. This means that, w_1 and $w_1(P_j)$ are perfect substitutes of each other. In other words, the owner can distribute the total wage of the low cost project manager equivalently in the form of a normal wage (w_1) and/or a preemption wage ($w_1(P_j)$) without altering anything else. At this point, for the contract to be uniquely defined, we assume that the owner will set $w_1(P_j) = 0$ and give all the wage to the low cost project manager in the form of a normal wage, $w_1 > 0$. Note that, when we later analyze the wages offered to lowest cost managers under a symmetric (Bayesian Nash) equilibrium, we will see that actually the lowest cost managers would have equilibrium exercise triggers equal to each other, i.e. $\underline{P}_j = P_1$ where $P_j \in (\underline{P}_j, \overline{P}_j)$, consequently it will not be possible for the lowest cost manager to be preempted by the other firm and to receive a preemption wage, $w_1(P_j)$.

The first-order condition with respect to w_1 gives,

$$\begin{aligned} & - \left(\frac{P_0}{P_1} \right)^\beta (1 - F(P_1|\theta_1)) + \lambda_1 \left(\frac{P_0}{P_1} \right)^\beta (1 - F(P_1|\theta_1)) - \lambda_2 \left(\frac{P_0}{P_1} \right)^\beta (1 - F(P_1|\theta_2)) = 0 \quad (49) \\ \Leftrightarrow & \lambda_2(1 - F(P_1|\theta_2)) = (\lambda_1 - 1)(1 - F(P_1|\theta_1)). \quad (50) \end{aligned}$$

The first-order condition with respect to w_2 gives,

$$\lambda_3 \left(\frac{P_0}{P_2} \right)^{-\beta} = \left(\frac{1}{q} - 1 - \lambda_2 \right) (1 - F(P_2|\theta_2)) + \lambda_1(1 - F(P_2|\theta_1)). \quad (51)$$

The first-order condition with respect to P_1 gives,

$$P_1 = - \frac{\beta + \hat{h}(P_1|\theta_1)}{1 - \beta - \hat{h}(P_1|\theta_1)} \left[\theta_1 + w_1 - \lambda_1 w_1 + \frac{\beta \frac{1-F(P_1|\theta_2)}{1-F(P_1|\theta_1)} + \frac{P_1 F'(P_1|\theta_2)}{1-F(P_1|\theta_1)}}{\beta + \hat{h}(P_1|\theta_1)} \lambda_2 (w_1 - \Delta\theta) \right]. \quad (52)$$

The first-order condition with respect to P_2 gives,

$$P_2 = - \frac{\beta + \hat{h}(P_2|\theta_2)}{1 - \beta - \hat{h}(P_2|\theta_2)} \left[\theta_2 + w_2 - \lambda_2 \frac{q}{1-q} w_2 + \frac{\beta \frac{1-F(P_2|\theta_1)}{1-F(P_2|\theta_2)} + \frac{P_2 F'(P_2|\theta_1)}{1-F(P_2|\theta_2)}}{\beta + \hat{h}(P_2|\theta_2)} \lambda_1 \frac{q}{1-q} (w_2 + \Delta\theta) \right]. \quad (53)$$

Eq. (50), together with a previous assumption ($F(P_j|\theta_1) \geq F(P_j) \geq F(P_j|\theta_2)$), implies either of the following cases to hold: (i) $\lambda_1 = 1, \lambda_2 = 0$ or (ii) $\lambda_1 - \lambda_2 > 1$.

Under the first case above, $\lambda_2 = 0$, the first-order condition with respect to w_2 , Eq. (51), implies that $\lambda_3 > 0$ and hence $w_2 = 0$.

Showing the similar result for the case where $\lambda_2 > 0$ is slightly more involved. In this case both incentive constraints bind. Substituting out w_1 from both, we obtain,

$$\left(\frac{P_1}{P_2}\right)^\beta \left[\frac{1 - F(P_2|\theta_1)}{1 - F(P_1|\theta_1)} - \frac{1 - F(P_2|\theta_2)}{1 - F(P_1|\theta_2)} \right] w_2 = \Delta\theta \left[1 - \left(\frac{P_1}{P_2}\right)^\beta \frac{1 - F(P_2|\theta_1)}{1 - F(P_1|\theta_1)} \right]. \quad (54)$$

We show $\lambda_3 > 0$ within a proof by contradiction. Assume initially that $\lambda_3 = 0$. Then, Eqs. (50) and (51) imply,

$$\left(\lambda_2 - \frac{1}{q} + 1\right) (1 - F(P_2|\theta_2)) = \lambda_1 (1 - F(P_2|\theta_1)) \quad (55)$$

$$\lambda_2 (1 - F(P_1|\theta_2)) = (\lambda_1 - 1) (1 - F(P_1|\theta_1)). \quad (56)$$

Dividing and rearranging results in,

$$\left[\frac{\lambda_1 \lambda_2 + \lambda_1 \left(1 - \frac{1}{q}\right) - \left(1 - \frac{1}{q}\right) - \lambda_2}{\lambda_1 \lambda_2} \right] \frac{1 - F(P_2|\theta_2)}{1 - F(P_1|\theta_2)} = \frac{1 - F(P_2|\theta_1)}{1 - F(P_1|\theta_1)} \quad (57)$$

Since $\lambda_2 > 0$ and $\lambda_1 - \lambda_2 > 1$ we obtain $\lambda_1 > 1$. This results in the term in square brackets of Eq. (57) to be strictly less than one, which in turn implies $\frac{1 - F(P_2|\theta_2)}{1 - F(P_1|\theta_2)} > \frac{1 - F(P_2|\theta_1)}{1 - F(P_1|\theta_1)}$. Using this result in Eq. (54), we find that the left hand side of Eq. (54) is negative, while the right hand side is clearly positive, a contradiction⁵². Therefore, λ_3 can not be zero also for $\lambda_2 > 0$. We can conclude that $\lambda_3 > 0$ holds in general and as a consequence $w_2 = 0$, which is summarized in the following proposition.

Proposition 10 *The limited liability constraint for a manager of a high cost (θ_2) project binds, i.e. $w_2 = 0$.*

Notice that, regarding the two incentive constraints, Eq. (50) implies that either Eq. (15) binds alone, or both Eqs. (15) and (16) bind simultaneously. First, remember that we obtain Eq. (54) when we equate the binding incentive constraints to each other by substituting out w_1 . However, we can see that the outcome of Prop. 10 is in conflict with Eq. (54). Therefore, Eq. (54) can never hold and we can conclude that Eq. (16) does not bind, $\lambda_2 = 0$, but Eq. (15) binds alone.

⁵² Note that for the special case of *no correlation*, $F(P|\theta) = F(P)$, Eq. (51) implies $w_2 = 0$ (Prop. 10), and we get a contradiction directly from Eq. (54) leading to the conclusion that the incentive constraint for a manager of a high cost (θ_2) project is never binding (Prop. 11).

Proposition 11 *The incentive constraint for a manager of a high cost (θ_2) project is never binding while the one for the low cost (θ_1) project always binds.*

So far the problem is simplified in that we are left only with the incentive constraint for a manager of a high cost (θ_2) project, together with the conditions $w_2 = 0$, $w_1(P_j) = 0$ and $w_2(P_j) = 0 \forall P_j \in (\underline{P}_j, \overline{P}_j)$.

Proof of Proposition 3. Making use of the Propositions (8) – (11), we simplify the optimal trigger levels P_1 and P_2 , expressed previously by Eqs. (52) and (53),

$$P_1 = -\frac{\beta + \hat{h}(P_1|\theta_1)}{1 - \beta - \hat{h}(P_1|\theta_1)}\theta_1 \quad (58)$$

$$P_2 = -\frac{\beta + \hat{h}(P_2|\theta_2)}{1 - \beta - \hat{h}(P_2|\theta_2)} \left[\theta_2 + \left(\frac{\beta + \hat{h}(P_2|\theta_1)}{\beta + \hat{h}(P_2|\theta_2)} \right) \left(\frac{1 - F(P_2|\theta_1)}{1 - F(P_2|\theta_2)} \right) \frac{q}{1 - q} \Delta\theta \right] \quad (59)$$

Finally, as Eq. (15) binds we get the optimal value of w_1 ,

$$w_1 = \left(\frac{P_1}{P_2} \right)^\beta \frac{1 - F(P_2|\theta_1)}{1 - F(P_1|\theta_1)} \Delta\theta \quad (60)$$

□

Proof of Proposition 4. Besides a few technical difficulties the solution of the agency problem in this setting is standard and we follow the derivations in Appendix A.3. of Grenadier and Wang (2005).

Given a realized θ , and a predetermined arbitrary exercise trigger $P_\theta > \theta$, the owner solves the optimization problem,

$$\max_{w_\theta, P_\theta} \int_{\theta_1}^{\theta_2} \left(\frac{P_0}{P_\theta} \right)^\beta \frac{1 - F(P_\theta)}{1 - F(P_0)} (P_\theta - \theta - w_\theta) dG(\theta) \quad (61)$$

subject to $w_\theta \geq 0$ for any θ , and $u(\theta, \theta) \geq u(\hat{\theta}, \theta)$ for any $\hat{\theta}$ and θ .

Note for the manager, who reports his privately observed component of the project cost is $\hat{\theta}$, and the true level of his privately observed cost is θ , we define his time-zero utility as,

$$u(\hat{\theta}, \theta) = \left(\frac{P_0}{P_\theta} \right)^\beta \frac{1 - F(P_\theta)}{1 - F(P_0)} (w_\theta + \hat{\theta} - \theta). \quad (62)$$

Following the derivations in Appendix A.3. of Grenadier and Wang (2005) we find that,

$$w_\theta = \left(\frac{P_\theta}{P_0}\right)^\beta u(\theta, \theta) = \int_\theta^{\theta_2} \left(\frac{P_\theta}{P(s)}\right)^\beta ds \quad (63)$$

and,

$$P_\theta = -\frac{\beta + \hat{h}(P_\theta)}{1 - \beta - \hat{h}(P_\theta)} \left(\theta + \frac{G(\theta)}{G'(\theta)}\right) \quad (64)$$

□

Proof of Proposition 5. We follow the derivations in Appendix A., the proof of Proposition 3 in Lambrecht and Perraudin (2003).

□

Proof of Proposition 6. We follow the derivations in Appendix A., the proof of Proof of Proposition 4 in Lambrecht and Perraudin (2003).

□

Proof of Proposition 7. We follow the derivations in Appendix A., the proof of Proof of Proposition 5 in Lambrecht and Perraudin (2003).

□

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